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# ReLU Strikes Back: Exploiting Activation Sparsity in Large Language Models

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Iman Mirzadeh, Keivan Alizadeh, Sachin Mehta, Carlo C Del Mundo,  
Oncel Tuzel, Golnoosh Samei, Mohammad Rastegari, Mehrdad Farajtabar  
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## Abstract

Large Language Models (LLMs) with billions of parameters have drastically transformed AI applications. However, their demanding computation during inference has raised significant challenges for deployment on resource-constrained devices. Despite recent trends favoring alternative activation functions such as GELU or SiLU, known for increased computation, this study strongly advocates for reinstating ReLU activation in LLMs. We demonstrate that using the ReLU activation function has a negligible impact on convergence and performance while significantly reducing computation and weight transfer. This reduction is particularly valuable during the memory-bound inference step, where efficiency is paramount. Exploring sparsity patterns in ReLU-based LLMs, we unveil the reutilization of activated neurons for generating new tokens and leveraging these insights, we propose practical strategies to substantially reduce LLM inference computation up to three times, using ReLU activations with minimal performance trade-offs.

## 1 Introduction

The enthusiasm for Large Language Models (LLMs) has spurred interest in AI across various domains [5, 9, 6]. However, the efficient utilization of LLMs faces challenges due to substantial computational and memory demands during inference [58, 39, 3]. To improve inference efficiency, techniques like quantization, speculative decoding, pruning, and weight sparsification have been explored [11, 48, 40, 51, 69, 19, 14]. Activation sparsity is particularly promising, offering a favorable trade-off between accuracy and speedup, especially on modern GPUs [49].

Notably, using Rectified Linear Unit (ReLU) activation induces sparse activations and has been widely adopted in prior works [26, 42, 46, 67]. We affirm this property using the OPT model [78] and measure the sparsity of the activations in the Feed Forward Network (FFN) layers (Fig. 1a). This sparsity substantially reduces computation (Fig. 1b), resulting in a 32% savings in FLOPS<sup>1</sup> (Fig. 1c).

However, a recent trend has emerged, favoring variations of ReLU that are smoother but more complex [27, 62]. These alternatives have gained popularity due to their slightly faster convergence and improved final accuracy [64]. For example, PaLM [9] and Llama models [71] adopt SiLU<sup>2</sup> [27, 16, 62], while MPT [54] and Falcon models [2] use GELU [27]. Nonetheless, as demonstrated in Fig. 1c, when we finetune several pretrained LLMs with different activation functions, their performance does not change significantly (within a specific model), while ReLU models require much less computation.

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<sup>1</sup>In this work, we use FLOPS as a proxy for inference efficiency. In Appendix C, we demonstrate that for LLMs with activation sparsity, FLOPS can serve as a good approximation of real-world efficiency due to the structure inherent in activation sparsity (e.g., skipping the entire row corresponding to zero activations).

<sup>2</sup>To be more precise, the mentioned models use SwiGLU activation function, but in this work, we focus on the gating module that uses SiLU (Swish) function.

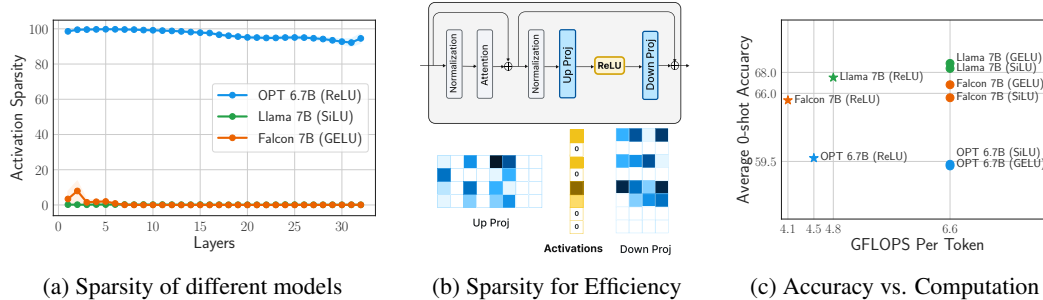


Figure 1: **(a)** Activation Sparsity of different pretrained models: ReLU-based OPTs show significantly higher sparsity. **(b)** Zeroed out entries after ReLU save compute in large semi-structured chunks (e.g., rows). **(c)** Comparison of inference efficiency and performance of the different models with different activation functions after fine-tuning: The choice of activation function does not significantly impact the accuracy, as any of GELU, SiLU, or ReLU can be used on all three models and achieve the same level of accuracy as the original activation function. However, using ReLU can provide an additional benefit of leading to activation sparsity and faster inference.

In this paper, we re-evaluate using ReLU for LLMs, emphasizing computational efficiency during inference. We demonstrate comparable performance with other activation functions when trained from scratch (Sec. 2) and highlight the benefits of fine-tuning existing pretrained LLMs with ReLU activations (Sec. 3). Moreover, built on the insights from our study, we propose several promising directions that can improve inference efficiency of LLMs (Sec. 4).

## 2 Does the Activation Function Matter?

This section first overviews our experimental setup, then, by training various models from scratch with different activation functions, we demonstrate that changing activation functions minimally impacts performance. However, the impact on inference efficiency is substantial.

**Experimental Setup.** We use open source pretrained models such as OPT [78], Llama (v1) [71], and Falcon [2] as they use different architectures and pretraining setup (e.g., attention/FFN structure/normalization, activation functions), allowing our study covers a wider range of models. We use the RefinedWeb dataset [57], for our pretraining in Sec. 2.1 and finetuning pretrained models in Sec. 3. We chose RefinedWeb because it is a high-quality subset of Common Crawl, which is often used in the pretraining phase of LLMs, including Llama, Falcon, and OPT. We also use the validation split of WikiText [52] for measuring the sparsity and recording preactivation distributions of various pretrained models. However, our conclusions hold for other datasets we have tested.

### 2.1 Training from scratch: performance and sparsity

While the previous literature suggests that non-ReLU variants can improve the performance of transformers [64, 55], we argue the impact is marginal at best. To support our claim, we train the OPT 1.3B model from scratch on a hundred billion tokens of the RefinedWeb datasets with different activation functions, including ReLU, SiLU, and GELU. All these activation functions can be viewed as  $f(x) = x \cdot \sigma(\beta x)$ , where  $\beta$  controls the gating part (smoothed cutoff threshold) of the activation function (see Fig. 2a). For  $\beta = 1$ , we will have SiLU( $x \cdot \sigma(x)$ ), and  $\beta = 1.7$  is a good approximation of GELU. Finally, as  $\beta \rightarrow \infty$ , the activation function becomes closer to ReLU. To further explore the spectrum of ReLU to SiLU we add another one with  $\beta = 8$ .

As shown in the bottom row of Fig. 2, the performance of the models is very similar when using different activation functions. This is consistent with the scaling laws literature ([36, 30]), which suggests that the performance of sufficiently large models trained on sufficiently large data depends heavily on compute and data, not architectural details.

While the performance levels of the different activations are similar, their activation sparsity levels differ. Here, we define sparsity as the average sparsity level across all layers for each model. Shown in Fig. 2c, as we transition from SiLU to ReLU (increasing  $\beta$ ), the sparsity also increases. This results

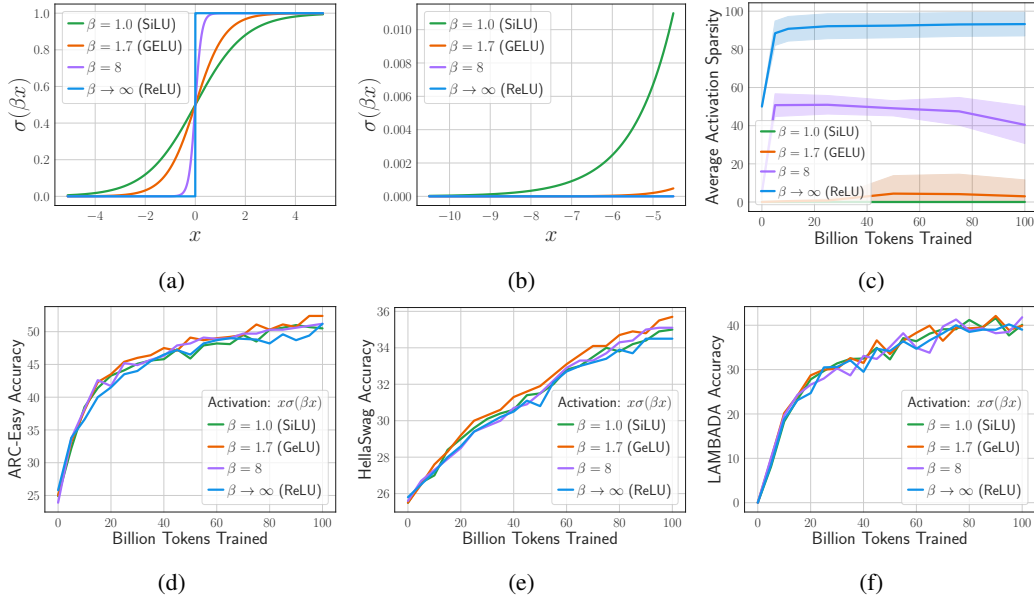


Figure 2: **(top)** (a) Shapes of different gating functions over  $[-5, 5]$ ; (b) Continuation of (a) where SiLU is comparably larger compared to others; (c) Sparsity of the FFN with different activations: increasing  $\beta$  increases sparsity. **(bottom)** when trained from scratch, OPT 1.3 B models using different activation functions achieve similar performance.

from the different gating thresholds, as ReLU drops significantly more values compared to GELU and SiLU (see Fig. 2b). In Appendix F.1, we illustrate the evolution of the pre-activation distribution throughout training.

Overall, the results support our initial claim: non-ReLU activations result in a negligible performance gain (if any) but a substantial loss in sparsity and efficiency. While, at times, the performance of GeLU or SiLU might be slightly higher, ReLU can match it with slightly longer training. We acknowledge that to compensate for the small gap in performance, we need to pay the one-time cost of longer training. However, in return, we get a significantly more sparsity.

### 3 Relufication: Improving Activation Sparsity of Pretrained LLMs

While in the previous section, we have seen that the performance does not depend on the activation function, we note that most of the available pretrained LLMs are trained with activation functions other than ReLU. Hence, to incorporate the computational benefits of ReLU activations at inference time, we perform various architectural surgeries and study the consequences of such changes.

We present our findings about incorporating ReLU activations into the pretrained LLMs, a process we refer to as *relufication*. More specifically, we show that replacing the activation functions of pretrained LLMs with ReLU is possible, and the performance can be recovered very rapidly during finetuning. We show these modifications, which are easy to implement, lead to more efficient models at inference time while maintaining comparable performance to the original pretrained models.

The process of *relufication* for different pretrained architectures is shown in Fig. 6. This process can be done in multiple stages, but here we focus on the first stage and postpone the second stage to Appendix D. The first and more intuitive stage replaces non-ReLU activations with ReLU in the FFN layer. For the Falcon and Llama models, this means replacing GELU and SiLU, respectively. We note that since OPT models already use ReLU activations, we keep those unchanged. After finetuning on 30 billion tokens of the RefinedWeb, Fig. 3 shows that the modified models have significantly more sparsity in their activations.

In addition to the drastic improvement in activation sparsity, we can make several notable observations. First, while the shape of preactivation depends on the pretraining dynamics and architecture, in Fig. 9,

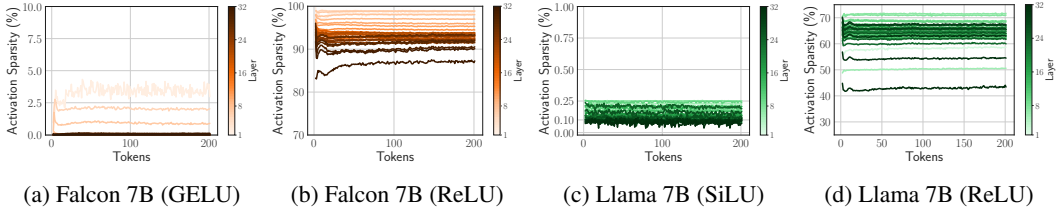


Figure 3: Activation sparsity of Falcon and Llama models improves significantly after *relification*.

Table 1: Comparing performance across several tasks: After the first stage of *relification*, the activation sparsity of models increases significantly, hence increased efficiency measured by FLOPS. Within each group, the performance levels are comparable.

Model	Sparsity (%)	FLOPS (G)	0-shot Accuracy (%)										MMLU (5-shot)
			Avg	Arc-E	Arc-C	Hellaswag	BoolQ	PIQA	LAMBADA	TriviaQA	WinoGrande	SciQ	
Falcon 7B (ReLU)	<b>94</b>	<b>4.1</b>	<b>65.2</b>	72.2	39.1	55.4	70.6	78.4	69.2	40.5	67.5	93.1	<b>27.9</b>
Falcon 7B (GELU)	1	6.6	<b>66.8</b>	74.6	40.2	57.7	73.5	79.4	74.5	40.4	67.2	94.0	<b>27.7</b>
Llama 7B (ReLU)	<b>62</b>	<b>4.8</b>	<b>67.1</b>	75.2	40.1	55.2	73.4	77.7	71.5	49.6	67.1	94.2	<b>34.7</b>
Llama 7B (SiLU)	0	6.6	<b>68.4</b>	75.5	42.1	69.9	74.8	78.7	73.1	49.9	69.8	95.4	<b>35.1</b>

we show that it does not change significantly during the relatively short finetuning stage. As a result, we can predict the activation sparsity before finetuning, knowing it will not change significantly. Later in Sec. E.3 we build on this observation and propose shifting the preactivation values before applying ReLU and further increasing the activation sparsity. The stability of the preactivation distribution may suggest that the behavior of the network does not change while creating sparse representations. Indeed, we show that after replacing the activation function with ReLU, finetuned models quickly recover their performance in Fig. 4. We believe optimizing this process even further (e.g., using better finetuning data) is an exciting follow-up direction.

## 4 Conclusion & Future Directions

In this study, we conducted a large-scale investigation of the activation functions, and we have shown that the choice of activation functions during pretraining and finetuning does not have a significant impact on performance while using ReLU can provide an additional benefit of leading to activation sparsity and more efficient inference.

To bridge the gap between existing pre-trained models and our work, we have *relified* several models to incorporate the ReLU activation function into the architecture of these already pre-trained models. We have shown that across several zero-shot and few-shot tasks, the ReLU-based LLMs perform similarly to their non-ReLU models at a significantly reduced computation.

As a stepping stone for future works, in Appendix D, we continue the *relification* process and we show that we can improve the efficiency of several LLMs even more. In addition, in Appendix E, we investigate two promising applications of activation sparsity in LLMs, and we show that we can leverage these sparse networks for faster inference via activation reuse and speculative decoding [40], in addition to exploring unconventional but promising activation functions for LLMs.

We believe our work is among the few studies that investigate changes in the architectural components of LLMs on a large scale. We hope our findings motivate the community to further investigate the advantages of well-structured activation sparsity, ultimately enhancing the efficiency of these models.

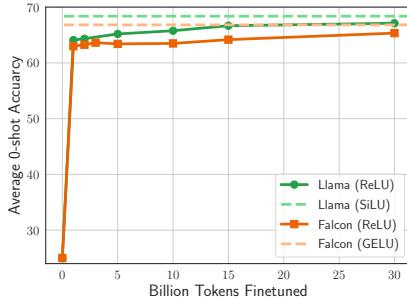


Figure 4: Evolution of zero-shot accuracy during finetuning: The model quickly recovers most of its lost performance due to the architecture surgery.

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## Appendix

The appendix is structured as follows:

- Appendix A expands the brief discussion of our experimental setup for this work.
- Appendix B provides a detailed discussion of the existing literature related to our work.
- Appendix C provides an important discussion on our *efficiency model*. We show that unlike the case for unstructured sparsity, activation sparsity is structured (e.g., skipping rows and columns in the calculation). Hence, in this case, FLOPS can be a reasonable proxy for efficiency.
- Appendix D continues the *relufication* process we studied in Sec. 3. We introduce the second stage of the relufication process to increase the sparsity, and we show that this has a negligible impact on performance.
- Appendix E discusses two promising directions based on the activation sparsity. We study the relationship between activation sparsity and relufication (Sec. E.1, and E.2). Moreover, we show that by understanding the preactivation distribution of the pretrained model, we can explore the idea of using unconventional activation functions such as *shifted relu* to improve the activation sparsity significantly (Sec. E.3).
- Finally, in Appendix F, we provide additional results on the preactivation distribution of OPT models trained from scratch and the viewing of the relufication process as a compression technique.

### A Extended Experimental Setup

**Models.** We use open source pretrained models such as OPT [78], Llama (v1) [71], and Falcon [2] as they use different architectures and pretraining setup (e.g., attention/FFN structure/normalization, activation functions), allowing our study to cover a wider range of models.

**Datasets.** We use the RefinedWeb dataset [57], for our pretraining in Sec. 2.1 and finetuning pretrained models in Sec. 3. We chose RefinedWeb because it is a high-quality subset of Common Crawl, which is often used in the pretraining phase of LLMs, including Llama, Falcon, and OPT. We also use the validation split of WikiText [52] for measuring the sparsity and recording preactivation distributions of various pretrained models. However, our conclusions hold for other datasets we have tested.

**Training and Finetuning.** For finetuning the pretrained models, we follow the original pretraining recipe, except we use a fixed learning rate of  $1.5e-5$  for Llama 7B, Falcon 7B, and OPT 6.7B models. In addition, we use the AdamW optimizer [50] for our finetuning with ZeRO stage 1 [60], where we shard the optimizer states across different GPUs. For pretraining OPT 1.3B models from scratch in Sec. 2.1, we follow the OPT training recipe.

**Evaluation.** For our *performance* evaluation, we use the few-shot tasks from Language Model Evaluation Harness [22]. We select these tasks such that they can measure various abilities of the models (e.g., reading comprehension, reasoning, etc.), and we aim to be consistent with other works in the literature to make the comparison easier. Consistent with the other sections, we compare activation sparsity as a measure of *efficiency*. Further details regarding the relationship between activation sparsity, FLOPS, and inference efficiency are discussed in Appendix C.

### B Related Works

**Activation Functions in Transformers.** The original Transformer architecture [72] was proposed with the ReLU activation function [21], following the popularity of ReLU at the time. Later, several studies aimed to improve the ReLU activation function by increasing its smoothness [27] and/or including parameterized gating mechanisms, such as GELU, SiLU, GLU, and SwiGLU [10, 62]. Earlier studies demonstrated the benefits of these alternatives to ReLU for transformers [64, 55], but on a small scale (e.g., they trained models up to a couple of 100M parameters with at most 35B tokens, while in this work, we train 1B parameter models on more than 100B tokens). However, we believe the impact of activation functions on performance is marginal, following scaling laws [36, 30], which state that architectural changes do not significantly impact performance.

**Activation Sparsity.** A body of prior research [42, 24, 68] has demonstrated that increasing sparsity can lead to reductions in both inference and training times. Dejavu [49] and [45] observed pronounced

sparsity in activations when using the ReLU function in feedforward layers. These studies propose that predicting this sparsity can further boost inference speeds. Similarly, [35] employed ReLU activations and introduced a controller to actively promote sparsity. Notably, these studies predominantly focus on networks employing ReLU activations, leaving out those with alternative activation functions. In contrast, our approach modifies networks by substituting other activation functions with ReLU. We then fine-tune these networks to achieve activation sparsity in the MLP layer post-ReLUfication. We further illustrate that inserting ReLU prior to the QKV and Feedforward layers can substantially reduce FLOPS, albeit at a minor cost to accuracy. Unlike the aforementioned studies, we do not utilize a sparsity predictor to minimize FLOPS.

**ReLU in Attention Mechanisms.** Beyond the activation function in the MLPs of large language models, a softmax activation is often employed within the attention module. Prior studies have indicated that it’s feasible to replace this softmax with ReLU without compromising accuracy [73, 66, 31]. This avenue of research is distinct from our approach of Relufication, which specifically focuses on activations preceding weight multiplications.

**Model compression for efficient inference** Quantization, pruning and distillation are the main three techniques for compressing neural networks [80]. Quantization has been used to reduce model size and faster inference [12, 48, 56, 11, 47, 43, 13, 38, 7, 74]. The quantized model occupies less space reducing the memory latency [20, 37]. Relufication is orthogonal to quantization and reduces the amount of memory required to be loaded and can further decrease the memory latency. Distillation [32, 29, 23, 53, 1] is another technique to train smaller models. This is orthogonal to using ReLU activations as any activation can be used in distillation methods. Sparsifying or pruning weights of neural networks [19, 34, 77, 69, 63, 51] can reduce computation and inference time. Weight sparsity is usually unstructured and hard to implement for hardware, but the sparsity induced by ReLU can easily be implemented as a matrix multiplication of non zero rows. Weight sparse models can be combined with our relufication for further decrease in compute.

**Mixture of Experts.** Mixture of Experts (MoE) LLMs usually subdivide the feed-forward layer into multiple experts. A router is then employed to selectively and sparsely activate these experts [65, 18, 70]. Similar to our work, MoE is a form of activation sparsity but in a group form and can be seen as a subset of sparse activation. Subsequent studies have further refined the inference and training methodologies for MoE models [59, 33, 75, 15, 41, 61, 81, 8, 25]. MoE can be also combined with Relufication, having sparsity inside FFN of each expert.

Another line of work is MoEfication of networks that have sparse activations by subdividing neurons [79]. Relufication can also help MoEfication be applicable to a wider range of networks by increasing sparsity of FFNs. For a more in depth review of mixture of expert models we refer the reader to [17].

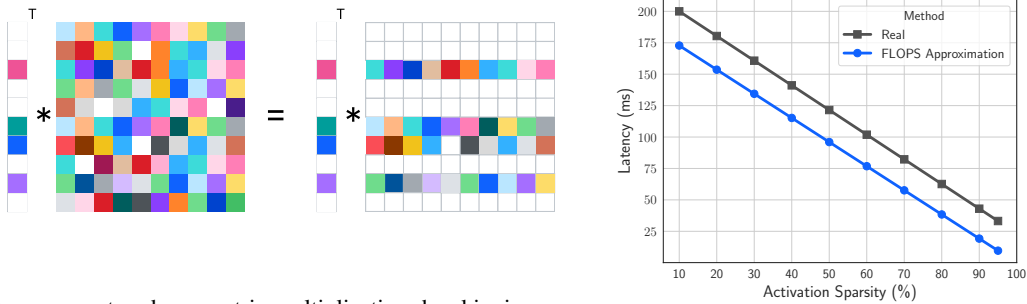
**Speculative Decoding and Sparsity.** Speculative decoding is a method that aims to improve model latency when faced with memory bandwidth constraints [44, 40]. It involves using a smaller model to predict the next tokens, with a larger model subsequently verifying multiple tokens in a single operation. In this work, we examine the direct effects of incorporating sparsity into speculative decoding. We show that adding sparsity can lead to performance improvements in speculative decoding. Additionally, we provide guidelines on selecting parameters for speculative decoding when sparsity is introduced.

## C Discussion on Activation Sparsity and Inference Efficiency

The primary motivation behind our work is to enhance *efficiency*, and we believe it is essential to provide a precise definition of this term. Throughout the main text, we predominantly use FLOPS as our efficiency metric. In this section, we argue why, in the presence of *activation sparsity*, FLOPS can serve as a suitable proxy for measuring various efficiency metrics.

Firstly, it is important to be reminded of the two major factors contributing to the efficiency of Large Language Models (LLMs): (1) the total amount of computation and (2) input/output (IO) transfer—i.e., transferring parameters from RAM to CPU/GPU for calculations. Notably, for today’s large models, factor (2) acts as the major bottleneck during the inference phase. We refer the reader to the detailed analysis by Liu et al. [49]. Ultimately, for a specific target device and assuming an efficient implementation, we believe that the most practical measure of efficiency is *latency* (e.g., the

average time to generate a token). However, each device possesses its unique properties, necessitating a more ubiquitous proxy metric.



(a) sparse vector, dense matrix multiplication: by skipping rows, we reduce both the weight transfer (i.e., loading these rows for computation) and computation (i.e., the result will be zero).

(b) Comparing FLOPS versus real latency for the OPT model (FFN).

Figure 5: For LLMs that have sparse activations, FLOPS is a good approximation of the real latency.

We argue that the way we calculated *FLOPS* in our paper and is greatly influenced by *activation sparsity* can reasonably approximate *efficiency*. Here are our reasons:

- **Reduced Computation:** As shown in Fig. 5a, with activation sparsity, we have a sparse vector-dense matrix multiplication at inference, while this will be a sparse-matrix-dense matrix multiplication during training. It is important to note that this is a semi-structured sparsity (unlike magnitude weight pruning), and we can leverage the fact that we are transferring weights in large chunks (i.e., *rows*). Modern accelerators already support sparse operations<sup>3</sup> and we can build on these existing tools.
- **Reduced IO Transfer:** During inference, weights need to be transferred to the device cache for computation (e.g., from RAM to CPU cache or GPU VRAM to GPU cache). This step constitutes the main bottleneck during token generation. For instance, approximately 99.3% of the total latency is attributed to IO, as indicated by Liu et al. [49]. However, by storing matrices in a row-major order, we can *skip loading* unnecessary rows as the output will be zero.

Overall, as depicted in Figure 5b based on the calculations by Liu et al. [49], we demonstrate that for the OPT model on an NVIDIA A100 GPU node, counting FLOPS provides a reasonable approximation to and is highly correlated with the time needs to generate tokens, especially, for LLMs with activation sparsity.

## D Extended Relufication: Pushing for More Sparsity

In Sec. 3, we replaced non-ReLU activations to gain more sparsity. This leads to the input of *down projection* layer being sparse, roughly 30% of the total computation. However, there are other matrix-vector multiplications in the decoder layer of transformers besides the down projection. For instance, before the *up projection* and *gate projections* of FFN layer, and *QKV projections* in the attention layer (see Fig. 6). Together, the mentioned matrix-vector multiplications consume about 55% of the total computation.

To this end, we utilize the fact that in modern transformer layers, the input to both the attention and FFN layers come from a normalization layer, e.g., LayerNorm [4] or RMSNorm [76]. These layers can be viewed as a specific form of MLP, where, instead of applying arbitrary learnable parameters, they learn to scale inputs. Therefore, we apply ReLU to obtain sparse activations after normalization layers which we call the *second stage* of relufication in Fig. 6, continuing the *first stage* of relufication we performed in Sec. 3.

Tab. 2 shows that different stages of the relufication process do not significantly reduce zero-shot accuracy while using significantly less compute. The sparsity is broken down into three categories:

<sup>3</sup>For example, both cuSPARSE on NVIDIA CUDA® and Accelerate on Apple devices.

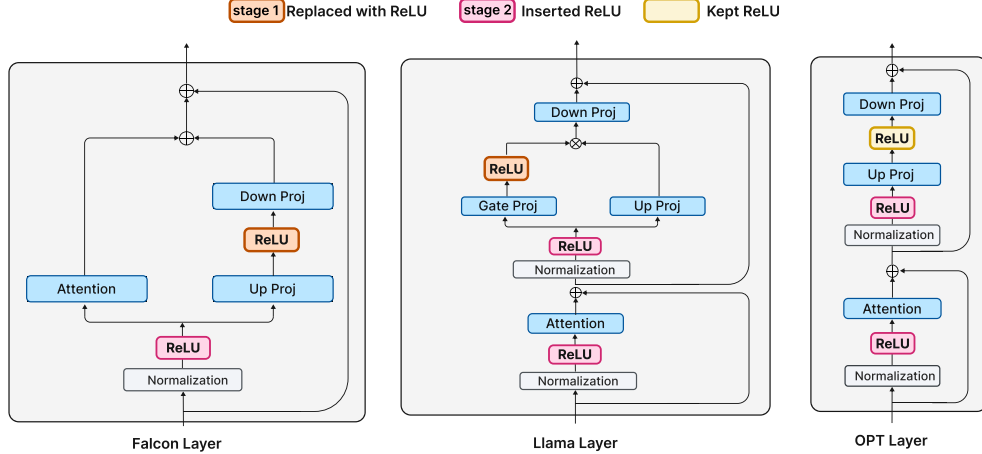


Figure 6: Architectural surgeries for *relification*. In stage 1 (c.f., Sec. 3), we keep the existing ReLUs (in the case of OPT) or replace the activation function between up projection and down projections from GELU (Falcon) and SiLU (Llama) to ReLU. In stage 2, we insert new ReLUs after normalization layers.

Table 2: Comparing zero-shot performance across several tasks: After *relification*, the activation sparsity of models increases significantly, hence increased efficiency measured by FLOPS. Within each group, the performance levels are comparable.

Model (stage)	Input Sparsity (%)			FLOPS (G)	Zero-Shot Accuracy (%)									
	QKV	DownProj	UpProj		Avg	Arc-E	Arc-C	Hellaswag	BoolQ	PIQA	LAMBADA	TriviaQA	WinoGrande	SciQ
OPT 1.3B	0	96	0	1.3	50.7	57.3	22.9	41.3	57.0	71.8	56.0	6.1	58.9	84.6
OPT 2.7B (s2)	50	96	35	1.1	53.1	60.3	26.8	44.9	55.4	73.9	57.6	12.4	59.6	86.7
OPT 2.7B	0	96	0	1.8	54.5	63.3	29.2	45.8	57.6	74.2	61.4	12.3	60.8	85.9
OPT 6.7B (s2)	50	97	40	2.8	58.6	66.5	32.2	49.1	63.0	76.4	63.3	23.8	63.1	90.3
OPT 6.7B	0	97	0	4.5	59.8	68.0	32.4	50.2	68.4	75.8	67.2	20.9	65.3	90.2
Falcon 7B (s2)	56	95	56	2.2	64.8	73.6	38.6	55.3	68.4	78.9	67.6	40.4	67.1	93.4
Falcon 7B (s1)	0	94	0	4.1	65.2	72.2	39.1	55.4	70.6	78.4	69.2	40.5	67.5	93.1
Falcon 7B	0	1	0	6.6	66.8	74.6	40.2	57.7	73.5	79.4	74.5	40.4	67.2	94.0
Llama 7B (s2)	51	65	67	2.9	66.4	73.8	39.6	54.8	69.9	77.9	70.7	48.5	68.6	93.8
Llama 7B (s1)	0	62	0	4.8	67.1	75.2	40.1	55.2	73.4	77.7	71.5	49.6	67.1	94.2
Llama 7B	0	0	0	6.6	68.4	75.5	42.1	69.9	74.8	78.7	73.1	49.9	69.8	95.4

up, down, and QKV projections. Notably, the input to QKV is less sparse than FFN projections, which opens an interesting avenue for future research. We note that the small gap in performance between the original vs. relified models may be partially due to the finetuning process and not necessarily the activation function. Our finetuning is applied only for 30B and 50B tokens for stages 1 and 2, respectively. Putting into prospect and comparing it with 1T tokens of Llama, for example, this is equivalent to 3-5% of the original training duration. As discussed in Sec. 2.1, according to the scaling properties of LLMs, the gap will be further bridged by additional finetuning steps.

We also assess the in-context learning ability of the relified models with the Massive Multitask Language Understanding (MMLU) [28] benchmark in Tab. 3. Our results show that when we augment the original LLMs with different activations and finetune, the few-shot performance does not change significantly either. Moreover, in Sec. F.2 we show that a larger but relified model outperforms the non-relified smaller model of the same FLOPS. Overall, the results affirm that the proposed relification procedure can decrease the inference FLOPS at various stages and rates while maintaining on-par performance on various tasks.

## E Applications

In this section, we discuss promising directions motivated by our investigation in Sec. 3. First, we introduce *aggregated sparsity*, showing that ReLU networks reuse previously activated neurons when generating tokens. Hence, we can leverage this to increase the generation speed. Next, we relate aggregated sparsity with speculative decoding to further improve speculative decoding’s inference

Table 3: MMLU five-shot accuracy. Models finetuned with different activation functions have similar performance.\* Denotes we replace the SiLU function in Llama’s SwiGLU activation function with ReLU.

Model	Activation	FLOPS(%)	Avg	Humanities	STEM	Social Sciences	Other
Falcon 7B	SiLU	100	26.4	24.8	27.4	27.2	26.2
Falcon 7B	GELU	100	27.7	28.1	26.0	28.0	29.4
Falcon 7B	ReLU	62	27.9	26.0	26.5	31.8	27.9
Llama 7B	SiLU*	100	35.1	37.9	30.2	37	37.1
Llama 7B	GELU	100	35.9	38.4	29.4	37.6	39.5
Llama 7B	ReLU	72	34.7	34.8	31.2	36.3	37.8

time. Finally, we briefly discuss a promising direction of using the *shifted ReLU* activation function to improve the sparsity further.

### E.1 Aggregated Sparsity: Reusing Previously Activated Neurons

A consequence of using only a small subset of neurons for each token is that if these neurons are shared to some degree, the model still does not use all of the neurons until many tokens are processed. We refer to this as *aggregated sparsity*, which we defined as the ratio of neurons that have not been used up to processing the first  $t$  token. Note that this metric will always be non-increasing. Intuitively, it measures the unused capacity of feed-forward neurons for processing a specific prompt.

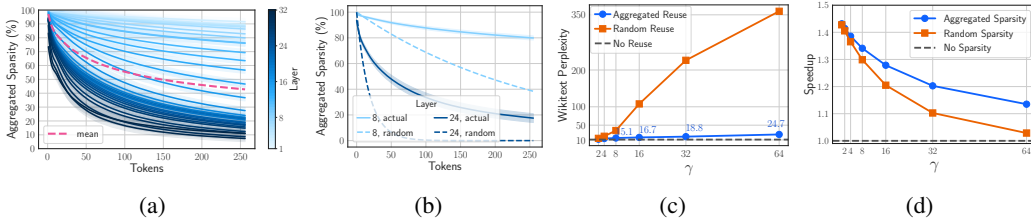


Figure 7: **(a)** Aggregated sparsity of different layers and their mean. **(b)** Aggregated sparsity during token generation and comparison with a random sparsity. **(c)** Perplexity, based on the number of tokens for which loaded weights from previous tokens are reused. The dashed line represents no reuse, the solid blue line shows the case with activation reuse according to aggregated sparsity, and the orange line depicts the perplexity when activations are reused according to a random sparsity. **(d)** The inference speedup of speculative decoding with aggregated sparsity and with random sparsity. Speedup equal to 1.0 is the standard version of speculative decoding.

Here in Fig. 7a we show that for the OPT-6.7B model, on average, about 50% of all the neurons will be unused across the first 150 tokens of prompts coming from the WikiText dataset. Our empirical results hold for other ReLU models and other datasets. Additionally, in Fig. 7b, we show that this pattern is far from random activation of neurons during the token generation with a rate equal to the average rate of activation usage per token. Let  $s_i$  be the activation sparsity of layer  $i$  averaged over all tokens. Then, the probability of an activation not used in generating the first  $t$  tokens in uniformly random selection is  $s_i^t$ . Fig. 7b shows this quantity for two layers  $i = 8, 24$  for the first 256 tokens in dashed line. It also shows the real (observed) number of activations being used in the solid line. The fact that the random aggregated sparsity (referred to as random sparsity) is lower than the observed aggregated sparsity (we refer to it as aggregated sparsity) shows a clear pattern of reusing activations.

We can benefit from the overlapping activations by utilizing previously loaded weights from the down projection layer for upcoming tokens. To test this, we initiate with reading 128 tokens. For the subsequent 128 tokens, we intermittently avoid loading new weights for every  $\gamma$  token. Using  $\gamma = 16$  as an example, tokens 129-145 are generated conventionally. However, for tokens 146-161, we retain the existing weight without introducing any new weight. This pattern continues, with every next set of  $\gamma$  tokens alternating between conventional generation and weight reuse. In Fig. 7c, we observe only a slight increase in perplexity when using this approximation to address the memory and I/O-intensive nature of LLM inference. This figure contrasts the perplexity obtained from reused activations and random selections. The reuse strategy aligns well with the baseline, whereas random

selection notably increases perplexity, highlighting the effectiveness of reusing the already loaded activations for subsequent tokens.

## E.2 Activation Sparsity and Speculative Decoding

As highlighted in Sec. E.1, activation reuse happens for multiple consecutive tokens. When multiple consecutive tokens are processed together, we can save the I/O (i.e., transferring weights to GPU/CPU as discussed in Appendix C) associated with activations that are not used in any of them. If the reuse was not happening, and the sparsity of all tokens was purely random, the aggregated sparsity would shrink exponentially and quickly diminish. Speculative decoding [44] is a related technique that uses a smaller model  $M_q$  to propose  $\gamma$  tokens and a larger model  $M_p$  to verify those tokens and select matching ones. It improves the runtime of the model by avoiding running  $M_p$  sequentially.

**Theoretical latency improvement.** To improve speculative decoding, aggregated sparsity can trim down the portion of the model that needs to be run. Instead of running the full model, only the non-sparse parts need to be evaluated, which will reduce I/O and compute latency. Suppose the average aggregated sparsity of  $M_p$  for  $\gamma$  tokens is  $\bar{s}_{\text{agg}}(\gamma)$ , and cost of running  $M_q$  over  $M_p$  is  $c$ . As per the primary text, token acceptances are assumed to follow an independent and identically distributed (i.i.d.) behavior. Denote  $\alpha$  as the expected probability of a token generated by  $M_q$  being approved by  $M_p$ . The following theorems hold:

**Theorem 1.** *The expected improvement factor in latency for speculative decoding with sparsity, over standard speculative decoding is  $\frac{c\gamma+1}{c\gamma+(1-\bar{s}_{\text{agg}}(\gamma))}$ .*

*Proof.* The amount of time required to run sparsified model is quantified as  $Tc\gamma + (1 - \bar{s}_{\text{agg}}(\gamma))T$ . It is the time of running a smaller model plus a larger model’s non-sparse portion. Run time of speculative decoding without sparsity is  $Tc\gamma + T$ . The number of generated tokens in both is the same. Therefore the relative speedup of using sparsity is given by:  $\frac{Tc\gamma+T}{Tc\gamma+(1-\bar{s}_{\text{agg}}(\gamma))T}$ .  $\square$

**Theorem 2.** *The expected improvement factor in latency, when combining sparsity with speculative decoding against normal (autoregressive) decoding using only  $M_p$ , is  $\frac{1-\alpha^{\gamma+1}}{(c\gamma+(1-\bar{s}_{\text{agg}}(\gamma)))(1-\alpha)}$ .*

*Proof.* Similar to theorem above  $Tc\gamma + (1 - \bar{s}_{\text{agg}}(\gamma))T$  gives the time required for sparse speculative decoding. According to the original paper, the standard speculative decoding yields an average of  $\frac{1-\alpha^{\gamma+1}}{1-\alpha}$  tokens generated per each run [44]. Thus, the anticipated run time when generating tokens with sparsity during speculative decoding becomes  $\frac{(c\gamma+(1-\bar{s}_{\text{agg}}(\gamma)))(1-\alpha)}{1-\alpha^{\gamma+1}}T$ . Given the runtime for producing a single token via an autoregressive approach is  $T$ , the inverse of this fraction gives the desired results.  $\square$

Fig. 7d compares sparse speculative decoding to the standard version for OPT 6.7B model. As a case study, for  $\gamma = 16$ , the sparse version has a 1.27x speedup over the standard speculative decoding. If the aggregated sparsity was random over different tokens, the speedup would have been only 1.20x. Note that even random sparsity will lead to speedup over standard speculative decoding. This further shows the value of relification. However, the speedup due to random sparsity would diminish quickly in comparison to aggregated sparsity as we go for larger  $\gamma$ . For example, for  $\gamma = 64$  the speedup is almost negligible, while the speedup for the aggregated sparsity is around 1.14x.

**Optimal  $\gamma$ .** The optimal  $\gamma$  for speculative decoding can be found by optimizing the speedup factor equation in Theorem 2. When sparsity is not present, the equation can be solved numerically, but for relified networks, the aggregated sparsity for different  $\gamma$ ’s will affect the final results. We have found optimal  $\gamma$ s based on  $\bar{s}_{\text{agg}}(\gamma)$  for OPT 6.7B and presented the results in figure 8a. The chosen  $\gamma$  for sparse speculative decoding is smaller than standard speculative decoding since higher  $\gamma$  will result in less sparsity. The gap in  $\gamma$  is always less than 20%. Also, in figure 8b, it can be seen for the specific case of  $\alpha = 0.8$ ,  $c = 0.02$ , the sparse speculative decoding has the highest speed-up factor over autoregressive at  $\gamma = 10$ s vs standard version’s optimal point which happens for  $\gamma = 12$ . Sparse speculative decoding at  $\gamma = 12$  is better than standard speculative decoding at  $\gamma = 12$ , while sparse speculative decoding at  $\gamma = 10$  beats both. Another observation from 8b is for the case of purely random sparsity, the benefit of sparse speculative decoding would diminish over standard speculative decoding in higher  $\gamma$ s. In contrast, the benefits of aggregated sparsity would last for larger values of  $\gamma$ .



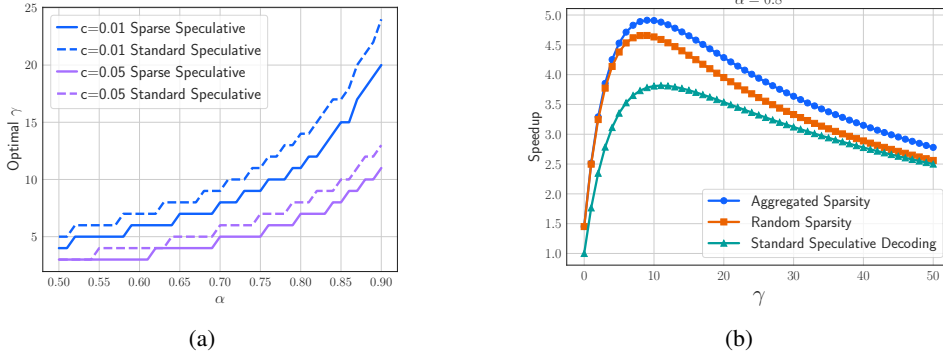


Figure 8: (a) optimal  $\gamma$  for sparse speculative decoding (b) speed up of sparse speculative decoding and standard speculative decoding over autoregressive decoding when  $\alpha = 0.8$  and  $c = 0.02$

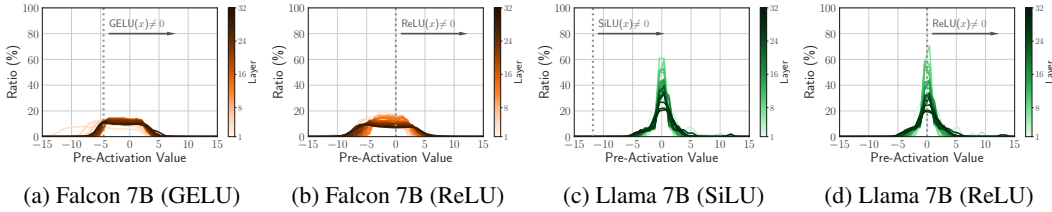


Figure 9: The preactivation distribution of pretrained models for Falcon and Llama does not change significantly during the short finetuning stage of relification. The dashed line shows the cutoff point before which the output is almost zero.

### E.3 The Shifted ReLU Activation

Our work in this section is motivated by the observation from Sec. 3, where, comparing Fig. 3d with Fig. 3b revealed that the relified Llama has much less sparsity (65%) than the relified Falcon model (95%). In addition, we build on two of our previous findings. First, the preactivation distribution of the relified Llama (Fig. 9c) includes a considerable mass after the cutoff value at zero. Second, the shape of the preactivation distribution does not change before and after the relification process (Fig. 9c and Fig. 9d).

Therefore, we may be able to shift the preactivation distribution to the left to put more volume before the cutoff at 0. To this end, for preactivation input  $x$ , rather than applying  $\text{ReLU}(x)$ , we use  $\text{ReLU}(x - b)$  where  $b \in \mathbb{R}$  is a constant scalar. We propose to set the value  $b$  based on the preactivation distribution. For instance, based on the distribution in Fig. 9d, setting  $b = 1$  and hence using  $\text{ReLU}(x - 1)$  as our activation function will result in dropping 95% of the preactivations and make it significantly sparser. Another benefit of this approach is simplicity, as this does not require changing the loss function or the training regime.

Figure 10a shows that the shifted ReLU activation function has on-par accuracy with the ReLU activation function. Moreover, similar to our observation in Sec. 3, the shifted ReLU activation quickly recovers the lost performance due to the drastic change of activation function, while it also maintains a very high-level activation sparsity during the finetuning stage. The gap between shifted ReLU and ReLU is wider in the early stages of training, and it narrows down when more tokens are seen.

A deeper investigation of ReLU-variants that can promote sparsity without sacrificing performance is an appealing future direction. Moreover, it will be interesting to study the impact of the shifted ReLU for stage-2 of our relification process where the sparsity level is usually not very high.

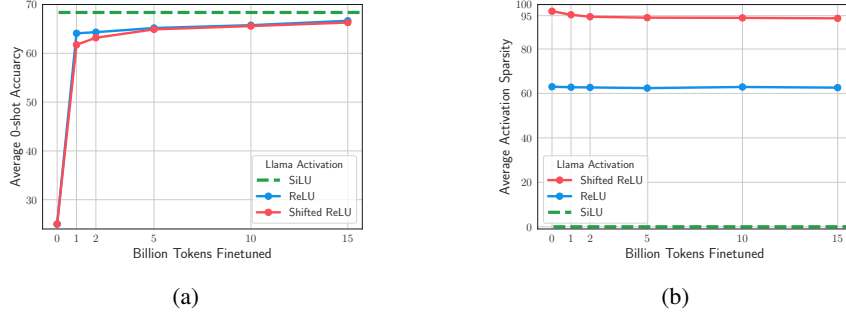


Figure 10: The effect of shifted ReLU on Llama model. **(a)** The performance is almost the same as the original ReLU. **(b)** Shifted ReLU (i.e.,  $\text{ReLU}(x - 1)$ ) is much sparser than the original ReLU.

## F Additional Results

### F.1 Pre-activation distribution of OPT models trained from scratch

The distribution of pre-activation inputs is suspected to be the primary factor determining the level of sparsity. As demonstrated in Sec. 3, the pre-activation distribution differs significantly for Llama and Falcon. One might question whether, when controlling for training data and optimization algorithms, the distribution shapes would vary. To investigate this, we trained OPT 1.3B models from scratch using our four activation function variants and illustrated the pre-activation distribution throughout training in Fig. 11. The distributions begin similarly but gradually diverge. Moving from SiLU to ReLU (increasing  $\beta$ ), the pre-activation distribution concentrates more around 0, almost forming a unimodal shape. Further exploration of the dynamics of pre- and post-activations and their correlation with efficiency and accuracy is an intriguing avenue for future research.

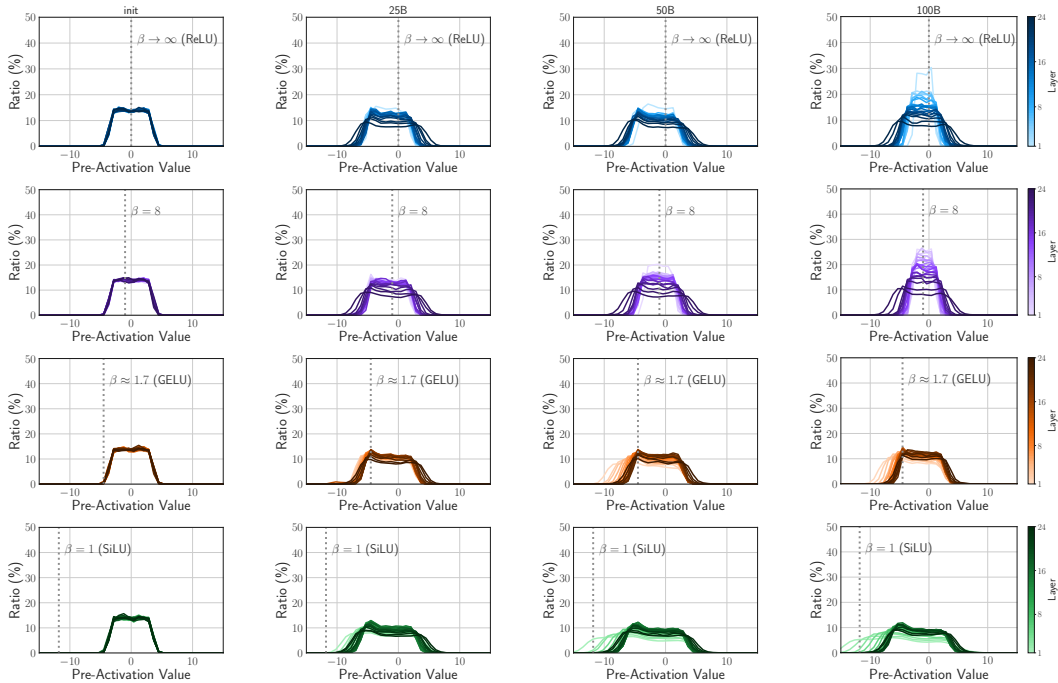


Figure 11: Pre-activation distributions of various OPT 1.3B models with all four types of activations trained from scratch at various number of seen tokens during training.

## F.2 Is a large relufied model better than smaller dense ones?

When it comes to deploying efficient models, one may naturally use an original smaller-size (dense) model. The argument would be the performance of the relufied larger model might be already equal to or less than the smaller dense model. To study the above question, we plotted the performance vs. efficiency of the original and the relufied OPT models in Fig. 12. Taking the relufied OPT 6.7B model as an example, it operates at 2.8 GFLOPs per token. Interpolating the blue line (that can be seen as a scaling plot of the OPT model), a dense model with equivalent FLOPs falls more than 2% short in zero-shot performance.

Similarly, compared to the relufied OPT 2.7B model, the equivalent (in FLOPs) dense model performs almost 2% lower. Indeed, the fact that the relufied models lie well above the scaling graph of the original OPT models, shows the effectiveness of relufication processes as a method to get better but more efficient models. As a side benefit, it makes the efficiency spectrum of the available LLMs more continuous. For example, consider a combination of hardware and use case that only allows deploying LLMs with lower than 3 GFLOPs during inference. Going with standard pretrained models, the only available option is OPT 2.7B with almost 1 GFLOPs, as the 6.7B does not satisfy the hardware constraint. In this situation, our relufied model not only falls in the limited inference budget but is also very close to the next largest available model in terms of accuracy. An exciting and timely direction for future research is finding methods, that, given an LLM (or a family of LLMs), are able to produce the best performing model matching the specified inference computation budget.

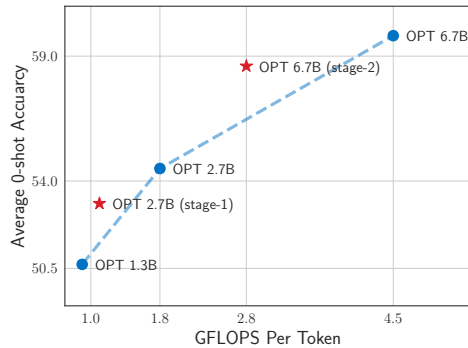


Figure 12: Performance of sparse large models vs. dense smaller models: The relufied large models (red stars) are above the scaling curve of original dense models (blue circles and dashed line).